# Geometry and Dynamics of Algorithms on the Quantum Information Space

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**Abstract**

Quantum computation and quantum information are known well to be very hot interdisciplinary fields [1], where a number of approaches have been taken. In a series of papers [2, 3, 4, 5], the author takes geometric and dynamical-system approaches to differential equations on the quantum information space (QIS) raised from algorithms in engineering.

One of the keys to the papers is the fact that the SLD-Fisher metric of the QIS is obtained from the natural Riemannian metric of the space of ordered tuples of multiqubits through a geometric reduction [2]. On the QIS, three types of geodesics are of much interest; the m-geodesics, the e-geodesics and the Levi-Civita geodesics. As a continuation of the paper [5] showing that the Grover search geodesic for a given ordered tuple of multiqubits is reduced to an m-geodesic on the QIS, this talk presents presents the classes curves which are reduced to the m-, the e-, and the Levi-Civita geodesics on the QIS. In particular, the curves related with the m-geodesics are discussed in detail.

Since the QIS consists of positive-(semi)definite matrices with unit trace, computer algebra would be worth applying to matrix computations on the QIS. Applicability of computer algebra will be discussed also in this talk.

**Keywords**
Quantum information space, Geodesics, Gradient flow, Algorithms

## References

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