Computing difference algebraic relations among solutions of linear differential equations

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While the classical Picard-Vessiot theory is concerned with the algebraic relations among the solutions of linear differential or difference equations, the parameterized Picard-Vessiot theory deals with algebraic relations among the solutions and their transformations under various operations like differentiation, shifting or scaling.

For example, the solution $J_\alpha(x)$ of Bessel's differential equation

$$x^2 y'' + xy' + (x^2 - \alpha^2)y = 0$$

satisfies the difference algebraic relation

$$x J_{\alpha+2}(x) - 2(\alpha+1)J_{\alpha+1}(x) + x J_\alpha(x) = 0.$$

This relation is witnessed by the associated Galois group, which is a difference algebraic groups, i.e., a group defined by difference equations. We will provide a computational perspective on how to compute these groups and find the corresponding difference algebraic relations.

References

[1] L. Di Vizio, Ch. Hardouin and M. Wibmer, *Difference Galois theory of linear differential equations*, Adv. Math. **260**, pp. 1-58 (2014)

[2] L. Di Vizio, Ch. Hardouin and M. Wibmer, *Difference algebraic relations among solutions of linear differential equations*, to appear in JIMJ