Towards a symbolic package for systems of nonlinear difference equations

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Difference algebra has been studied in analogy to differential algebra. However, concepts such as characteristic sets for differential systems have not been developed in the same generality for difference systems yet. In particular, methods such as the Rosenfeld-Gröbner algorithm, regular chains and Thomas decomposition for differential systems are not available for difference systems. Among the many applications of difference algebra is, e.g., the consistency analysis of finite difference schemes for partial differential equations.

This talk presents results of trying to transfer the concept of differential Thomas decomposition to systems of nonlinear difference equations and develop a symbolic package for systems of nonlinear difference equations. It reports on joint work with Vladimir Gerdt.

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