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Formal Power Series Solutions of First Order Autonomous Algebraic Ordinary Differential Equations

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Let $\mathbb{K}$ be an algebraically closed field of characteristic zero. Given a first order autonomous algebraic ordinary differential equation, i.e. an equation of the form

$$F(y, y') = 0 \text{ with } F \in \mathbb{K}[y, y'],$$

we present a method to compute all formal power series solutions. Furthermore, by choosing for instance $\mathbb{K} = \mathbb{C}$, the computed formal power series solutions are indeed convergent in suitable neighborhoods.

We follow the algebro-geometric approach by Feng and Gao [2] and consider y and y' as independent variables, let us say y and z. Then F implicitly defines an affine plane curve where local parametrizations can be computed, see e.g. [3]).

We show a sufficient and necessary condition on such a local parametrization to obtain a formal power series solution of the original differential equation by substitution. Moreover, we present a polynomial-time algorithm for computing all the initial tuples, i.e. the first two coefficients of a formal power series, which can be extended to a solution. By choosing a particular initial tuple, a second algorithm determines the coefficients of all solutions starting with this initial tuple up to an arbitrary order.

A full version [1] has been submitted to a journal and is online available.

Keywords: Algebraic autonomous differential equation, algebraic curve, local parametrization, formal power series solution

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