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Group Classification of ODEs: a Challenge to Differential Algebra?

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One of the most prominent application of differential algebra is algebraic analysis of determining system of partial differential equations for infinitesimal symmetry generators. It provides receipts and software tools to compute the integrability conditions, to simplify (e.g. to interreduce) the system, to determine a dimension of its space, to construct the abstract Lie algebra for the symmetry generators, to apply the Lie symmetry algebra for ordinary differential equations (ODEs) to detect their linearizability [1] by point transformations. The problem of group classification for differential equations was first posed by the Norwegian mathematician Sophus Lie, the inventor of the concept and theory of continuous groups and their application to differential equations [2]. Lie began to solve the group classification problem for the second-order ordinary equation $y'' = f(x, y, y')$ and proved that this class of equations admits no more than an eight-parameter transformation group on the plane with the maximum size of the group is reached iff the equation is linear or equivalent to the linear one. The Russian mathematician Lev Ovsyannikov [3] proposed the equivalence transformation (ET) method for group classification and later [4] applied it to the ODE of form $y'' = f(x, y)$. The ET method is based on the fact that equivalent equations admit similar groups and ET is a similarity transformation. The problem of group classification admits reformulation as an elimination problem in differential algebra. However, even reproduction of the results, obtained in [4] by hand computation, seems to be too hard for the modern differential elimination tools. In the talk we discuss both the pure mathematical and computational issues of the group classification for ODEs.

Keywords: Differential Algebra, Group Classification, Ordinary Differential Equations

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