Applications of Computer Algebra – ACA 2019
Montréal, Canada | July 16-20, 2019
École de technologie supérieure

A Maple package for solving algebraic differential equations by algebro-geometric methods*

Johann J. Mitteramskogler[1]***, Wolfgang Schreiner***[1]***, Franz Winkler***[1]
[johann.mitteramskogler@risc.jku.at]

[1] Research Institute for Symbolic Computation (RISC), Johannes Kepler University Linz, Linz, Austria

AGADE (Algebro-Geometric methods for solving Algebraic Differential Equations) is a software package for computing various types of symbolic solutions for algebraic differential equations (ADEs). This project is still in an early stage of development. We plan to present solution algorithms for finding rational general solutions of first-order (non-linear) ordinary ADEs based on the approaches in Feng and Gao [1, 2] and Ngô and Winkler [3, 4]. The computation of this type of solution, which must contain a transcendental constant, requires knowledge of explicit degree bounds for rational invariant algebraic curves in the case of non-autonomous ADEs. Such a bound is, however, only known in the generic non-dicritical case [5]. An algorithmic way of completely deciding the existence of—and in the positive case, computing—rational general solutions is known for the subclass where the transcendental constant appears rationally [6]. An implementation of the latter method will be part of a subsequent release, however. Later versions of the package will also provide methods for other solution types such as algebraic, radical or formal power series solutions, as well as semi-algorithmic procedures for partial ADEs and systems thereof. An overview can be found in Grasegger and Winkler [7, 8]. All solution methods utilize an approach known as the algebro-geometric method for solving ADEs [9]. A crucial step in this approach is the parametrization of an algebraic variety obtained from the differential equation, where the type of the parametric equations follows from the solution class one is interested in. Given a suitable parametrization of this variety, one obtains an associated system of differential equations whose solution set is in one-to-one correspondence with the solutions of the original ADE. Due to its special form, solutions of the associated system can be computed by well-known methods and are then transformed back to solutions of the original differential equation. This software package is developed for the widely used computer algebra system Maple†.

Keywords
Algebraic differential equation, rational general solution, symbolic computation, parametrization, software package

*This project is supported by the Austrian Science Fund (FWF): P31327-N32
†Maple (2018). Maplesoft, a division of Waterloo Maple Inc., Waterloo, Ontario.

References

[1] R. FENG; X. S. GAO, Rational general solutions of algebraic ordinary differential equations. In Proceedings of the 2004 International Symposium on Symbolic and Algebraic Computation, 155–162, Santander, Spain, 2004.

[2] R. FENG; X. S. GAO, A polynomial time algorithm for finding rational general solutions of first order autonomous ODEs. *Journal of Symbolic Computation* **41**(7), 739–762 (2006).

[3] L. X. C. NGÔ; F. WINKLER, Rational general solutions of first order non-autonomous parametrizable ODEs. *Journal of Symbolic Computation* **45**(12), 1426–1441 (2010).

[4] L. X. C. NGÔ; F. WINKLER, Rational general solutions of planar rational systems of autonomous ODEs. *Journal of Symbolic Computation* **46**(10), 1173–1186 (2011).

[5] M. M. CARNICER, The Poincaré problem in the nondicritical case. *Annals of Mathematics* **140**(2), 289–294 (1994).

[6] N. T. VO; G. GRASEGGER; F. WINKLER, Deciding the existence of rational general solutions for first-order algebraic ODEs. *Journal of Symbolic Computation* **87**, 127–139 (2018).

[7] G. GRASEGGER; F. WINKLER, *Symbolic solutions of first-order algebraic ODEs*. J. Gutierrez, J. Schicho, M. Weimann (eds.), LNCS volume 8942, 94–104, Springer International Publishing, 2015.

[8] G. GRASEGGER; F. WINKLER, A solution method for autonomous first-order algebraic partial differential equations. *Journal of Computational and Applied Mathematics* **300**, 119–133 (2016).

[9] F. WINKLER, The algebro-geometric method for solving algebraic differential equations — A survey. *Journal of Systems Science and Complexity* **32**(1), 256–270 (2019).