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A differential algebra approach to parameter identifiability in ODE models

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We study structural identifiability of parameterized ordinary differential equation models of physical systems, for example, systems arising in biology and medicine. A parameter is said to be structurally identifiable if its numerical value can be determined from perfect observation of the observable variables in the model. Structural identifiability is necessary for practical identifiability.

The question of parameter identifiability is of great importance in modeling, e.g. in biological systems. Recent work studies identifiability in oncology ([1]), phylogeny ([2]), and cardiovascular models ([3]). Various techniques have been used to study identifiability, and the use of differential algebra in particular extends back 30 years (see, e.g., [4]).

We study structural identifiability via differential algebra. In particular, we use characteristic sets. A system of ODEs can be viewed as a set of differential polynomials in a differential ring, and the consequences of this system form a differential ideal. This differential ideal can be described by a finite set of differential equations called a characteristic set. The technique of studying identifiability via a set of special equations, sometimes called "input-output" equations, has been in use for the past thirty years. However it is still a challenge to provide rigorous justification for some conclusions that have been drawn in published studies.

Our main result is on linear systems. Identifiability in linear systems is a topic of current interest (see [5], [6], [7], [8], [9], [10]). We show that for a linear system of ODEs with one output, the coefficients of a monic characteristic set are identifiable. This refines results presented in [10] and [6]. Our result can be generalized, with additional hypotheses, to nonlinear systems with multiple outputs.

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Keywords
Identifiability, Mathematical Biology

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