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Order bounds for differential elimination algorithms

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Differential elimination is the process of eliminating a fixed set of unknown functions from a system of differential equations in order to obtain differential consequences of the system that do not depend on the eliminated functions. The Rosenfeld-Gröbner algorithm, which first appeared in [1], approaches the problem of differential elimination through differential decomposition, that is, by breaking down the original system of differential equations into a collection of simpler systems that can be more easily studied. Properties of these simpler differential systems (for example, whether or not they depend on the to-be-eliminated functions) can then be used to determine information about the original system of differential equations.

In this talk we will discuss the complexity of the Rosenfeld-Gröbner algorithm in terms of the orders of the derivatives that appear in the algorithm. The first such complexity bound was found in [2] for the case of a single derivation. In [3] this was extended to the case of an arbitrary number of derivations. This new upper bound is made possible by associating to the algorithm certain antichain sequences that could be bounded using new results in [4]; the upper bound is then given in terms of the length of these antichain sequences. Also presented is a refined bound for the case of two derivations. The talk is based on joint work with Alexey Ovchinnikov and Gleb Pogudin.

Keywords
Partial Differential Equations, Differential Elimination, Rosenfeld-Gröbner Algorithm

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