14th Computer Algebra Seminar

on Computer Algebra Systems in Teaching and Research

LIT / Joint Institute for Nuclear Research
Dubna / Russia , June 2 - 3, 2011

Solution for certain classes of PDEs with the Method of Inverse Differential Operators

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Abstract.

The *Method of Inverse Differential Operators* (well established for ODEs) is extended in order to be applied to non-linear PDEs. Implemented in *Mathematica* a wide class of non-homogeneous PDEs can be solved. One restriction is that the PDEs must be linear and the coefficients of the operator differential polynomial $\chi(\mathcal{D}_{x_1}, \mathcal{D}_{x_2}, \ldots)$ have to be constant.

In order to obtain the *homogeneous* solution u_h it is essential that χ can be decomposed into linear factors $(\alpha_1 \mathcal{D}_{x_1} + \alpha_2 \mathcal{D}_{x_2} + \ldots + \alpha_n \mathcal{D}_{x_n} + \gamma)^\kappa$ (of multiplicity κ) for a subset of n independent variables $\{x_1, x_2, \ldots x_n\} \in \{t, x, y, z, \xi, \eta, \zeta\}$.

To determine the *particular* solution u_p for non-homogeneous PDEs there exist some restrictions as regards to the non-homogenuity $\phi(x_1, x_2, \ldots)$. To obtain the solution u_p it is essential that a series of replacement rules for the differential operators $\mathcal{D}_{x_i}$ can be applied to the non-homogenuity $\phi(x_1, x_2, \ldots)$ which causes some restrictions on the functional form of ϕ. Thus, the non-homogenuity must be either an arbitrary monomial $\phi_1 = M(x, y, z, \ldots) = \sum_{i=1}^{k} \alpha_i\, x^{k_i}\, y^{m_i}\, z^{n_i} \cdot \ldots$ or an exponential, trigonometric (sin|cos) and hyperbolic (sinh|cosh) function ϕ_2 with linear arguments. Moreover, any *multiplicative* combination of an exponential ϕ_1 with trigonometric|hyperbolic functions ϕ_2 or the *sum* of all these functions including monomials is admitted.

Finally, the special case of 2nd order PDEs subject to *initial conditions* $u(x_1,0) = \phi(x_1)$ and $u_{x_2}(x_1, 0) = \psi'(x_1)$ is treated. The only restriction is that the differential operator polynomial $\chi\left(\mathcal{D}_{x_1}{}^2, \mathcal{D}_{x_2}{}^2, \ldots\right)$ must be factorizable into two linear factors in order to determine u_h . The type of PDEs to be considered are the wave equation and the Laplace equation.

Only in a few cases *Mathematica*'s built-in routine DSolve is able to find a solution.